Serial Probability Recursion as a Method to Assess Manhattan Rental Rate Turning Points

Michael Colacino

I. Abstract

The paper applies an algorithm for turning point prediction originally developed by Neftçi (for assessment of the business cycle) to the New York City commerical office market. Using office employment data as an indicator, the serial probability recursion algorithm attempts to predict peaks and troughs in asking rents (as provided by CoStar), and is tested against rental data from 1996 to 2016. The algorithm provides a continuous probabilistic assessment of the likelihood of a turning point, which can be monitored (as new data becomes available) to assess tenant strategies in an evolving market. Three different methods of computing the algorithm's transition probabilities are examined, two from the literature and one unique to the paper. These approaches are compared using diagnostic measures from the literature. Although the algorithm does not provide significant (multiple-quarter) "look ahead" power, its results are good at calling the turning point in real time, thereby providing strategic guidance at the ambigous and essential instant of a market change.

II. Introduction

Commercial office rental rates in Manhattan change gradually over time, with typical cycles taking between four and twenty years. Like all financial time series, the rental rate cycle has noise in the data, so it it sometimes difficult to assess a turning point. Understanding the point at which the series inflects is extremely useful in planning the long term process required to find, negotiate for, and construct office space, particularly for larger tenants.

The economic drivers of the commercial office market are complex. Construction/development time lags, regulatory restrictions, and the potential for the nearly instantaneous influx of sub-lease space all complicate the understanding of supply. Macro-economic demand shocks (like the dot-com bubble and the global financial crisis) combine with non-stationary trends in space utilization and a non-linear relationship between new jobs and space absorption to make understanding the market's demand for space even more difficult.

Despite these complications, it should be possible in theory to develop demand-side leading indicators that precede the slow-motion changes in supply, or at the minimum to confirm the observation of turning points in rental rates. One method is described in this paper.

In the case of the business cycle (the predominant focus of the research in the bibliography) the key goal is an early prediction of a turn in the series; in the case of the real estate cycle, strategic planning could also be aided by timely confirmation that the cycle has turned.

1. Turning Points in Economic Research Literature

The literature on turning points in a general economic context is very far-ranging. A survey of a variety of methods can be found in Andersson, Bock, & Frisén, 2005, which evaluates the relative performance of techniques based on regime-switching hidden Markov models, piecewise linear models, and non-parametric linear models. A table comparing of the traits of these techniques in displayed on p. 467 of Andersson.

Many of the methods of turning point prediction stem from work by Arnold Zellner and his collaborators employing autoregressive approaches. See for example Zellner, Hong, & Min, 1991. In Hamilton, 1989 the author employs a Markov switching regression approach, and this approach is adopted in several subsequent papers.

As Chin et al point out, turning point analysis is important because many traditional techniques of forecasting (such as autoregressive methods like VAR) rely heaviliy on recent prior periods for prediction of future values. These methods generally make their largest errors at turning points, Chin, Geweke, & Miller, 2000 citing McNees, 1992. Chin et. al employ a probit model to predict turning points in the unemployment rate.

As Frisen, 1994 points out, some statistical models provide very good results between turning points, but poor results in predicting them. She and other authors postulate that there is an asymmetry in the business cycle, with stochastic behavior that differs between up and down cycles, which would explain the poor performance of models that don't provide different regimes for upward and downward movement of the underlying economic process. Friśen calls her work "statistical surveillance," and says:

> *In many different areas there is a need of continual observation of time series, with the goal of detecting an important change in the underlying process as soon as possible after it has occurred. p. 3.*

In the case of real estate leasing markets, this type of surveillance is almost as useful as a truly predictive model. A great deal of benefit is achieved by replacing linear extrapolation of rental trends with an assessment of whether a peak or trough has been encounterd. Knowing this informs the "go/no-go" decision faced by tenants as they consider when to strike with a renewal, a new lease, or a sublease and relocation.

The tool which is the focus of this paper is the **Serial Probability Recursion** or **SPR** method. This technique is described in Neftçi, 1982, Palash & Radecki, 1985 and in Diebold & Rudebusch, 1989 and contrasted with vector autoregressive (VAR) models in Webb, 1991 and Del Negro, 2001. Webb finds that VAR methods and SPR methods both work as well as expert predictions which combine extensive analysis, and both methods are roughly equivalent in their predictive power to expert predictions that feature extensive analysis of several hundred time series.[1] This technique is applied to business cycles in a variety of different countries by Niemira, 1991.

Neftçi uses this method to study the business cycle, employing the NBER Leading Indicator series. As he describes the model:

> *...we assume that the forecaster is observing a process {X_t} whose probability structure changes abruptly at some random time period. The popular notion of a 'downturn' is assumed to occur during this switch. The decision-maker's objective is to predict in some 'optimal' sense when this switch will occur. The change in probability structure is not directly observed; as a result observations on {X_t} will have to be used to make inferences on whether the economy has entered a new regime or not. Thus, the problem is to obtain a prediction rule which signals the switch in distribution as soon as possible, given that the number of false alarms are kept at a minimum.*

One of the advantages of this method is that it produces a probability of turning point occurence. As that probability gets above some threshold, a user could choose to act. This method lends itself to strategic decision making, particularly when combined with a broader set of analyses of the state of a real estate market.

The balance of this paper will explain the application of this method to real estate rental data and office-using employment.

1. Webb, 1991, p. 120

2. Data to be Examined

Rather than examining the business cycle, this paper will focus on Manhattan office rents over the period from 1996 to the third quarter of 2016. These asking rents (derived from CoStar data) are CPI adjusted for the analysis:

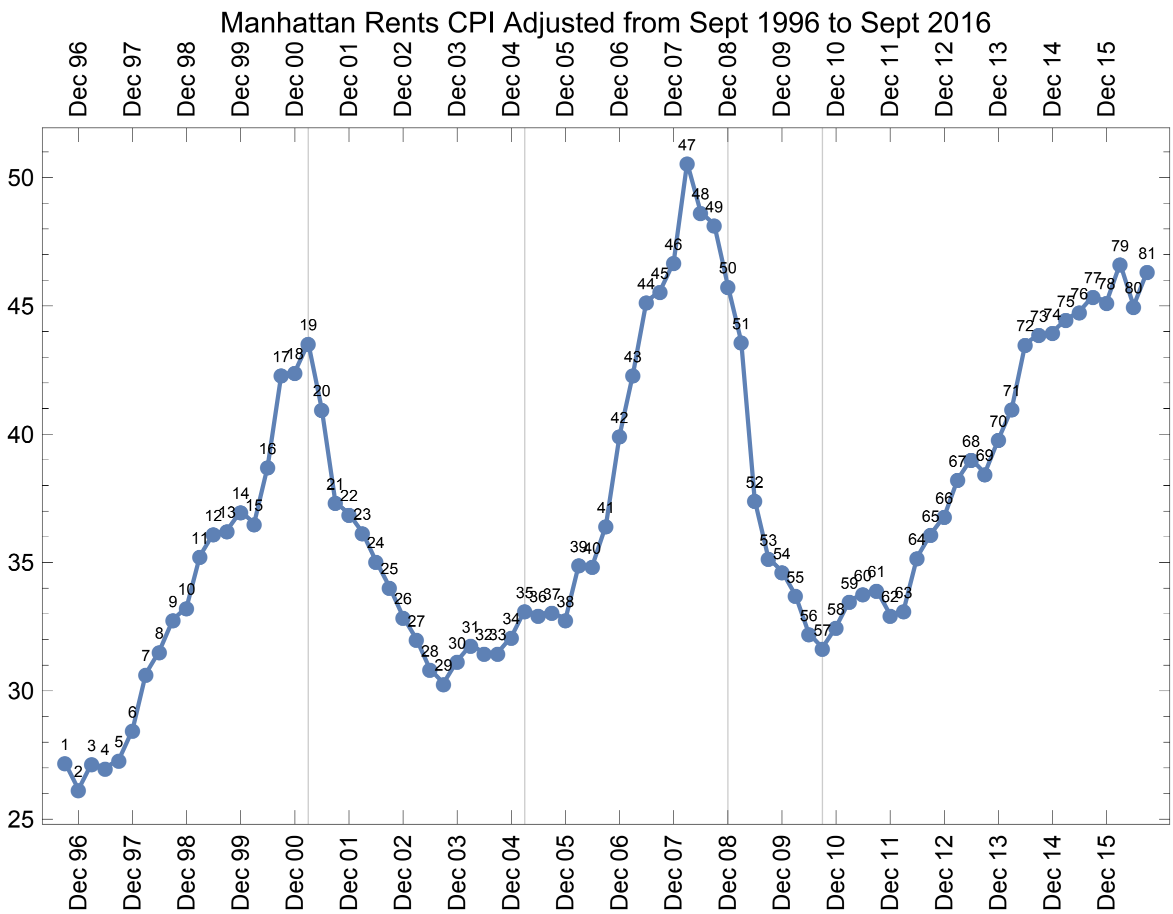

Figure 1: Manhattan Rents

As an indicator series the paper employs Manhattan office employment data from the Bureau of Labor Statistics. Figure depicts changes in office employment in Manhattan from 1996 to 2016. Peaks are observed in 2000 and 2007, with a possible peak in 2013. Troughs occur in 01 and 09. Since this figure is year over year percentage changes, all data above 0 indicates a quarter experienced positive office employment growth.

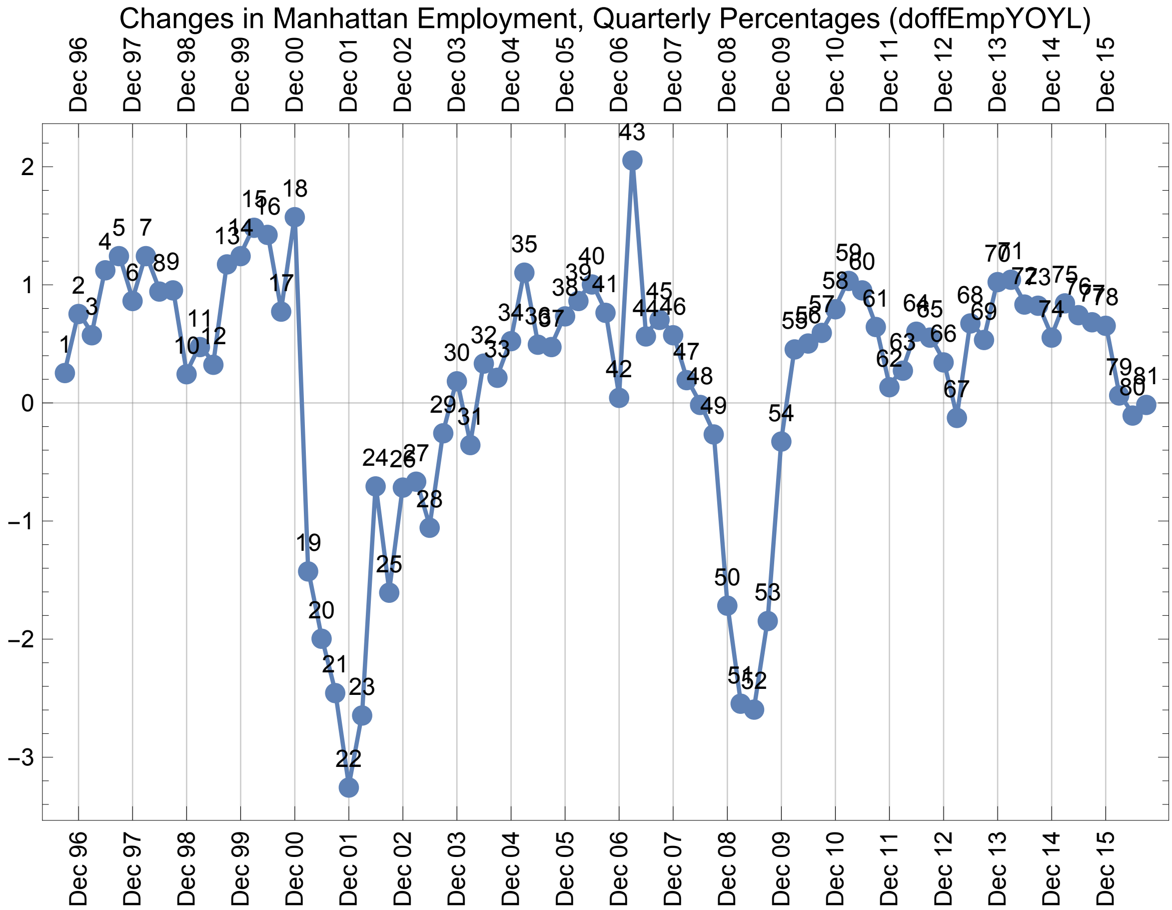

Figure 2: Changes in office employment YOY

In order to analyze this data and determine if the (index) employment series is useful in either anticipating or confirming turning points in the rental series, the Neftçi algorithm will be applied.

3. Neftçi Algorithm Description

Neftçi constructs his algorithm in several steps. First, the indicator data is separated into *up and down regimes* with the goal of assessing the likelihood that any new observation falls into one or the other regime. Neftçi smooths his data to obtain two probability distributions from which to make comparisons against new data. He labels them F^0 (upturn) and F^1 (downturn) densities. The computation of these densities are examined in greater detail in section **I.4**.

Second, Neftçi proposes a state-change or *a priori probability function*. He constructs the probability using a smoothing function examined in greater detail in section **I.5**. In his paradigm, there is an increase in the probability of a transition as the cycle progresses. This concept is not universally accepted by authors who wrote on this topic subsequent to Neftçi, as also discussed in greater detail in section **I.4** below.

These two components are then used in a recursive function to determine the probabity of a downturn:[2]

$$\begin{aligned}\pi_{k+1} = {} & [\pi_k + P(Z=k+1|Z>k)(1-\pi_k)]p^1_{k+1} \\ & /\{[\pi_k + P(Z=k+1|Z>k)(1-\pi_k)]p^1_{k+1} \\ & +(1-\pi_k)p^0_{k+1}[1-P(Z=k+1|Z>k)]\}\end{aligned}$$

Equation 1: Neftçi formula

In this formula the probability π of a turning point in a given (*k+1*) period is a function of:

a) the (recursively-defined) prior period's probability (π_k),
b) the *a priori* probability (the probability that the turning point *Z* will occur in period *k+1* given that it has not already occurred), and
c) the conditional probabilities p^0 and p^1 that determine the "unusualness" of the current move based upon the historical nature of upward and downward moves. These latter probabilites are determined through *the* F^0 and F^1 density functions introduced above.

4. Diebold & Rudebusch

Diebold & Rudebusch, 1989 follow Neftçi, employing slightly different nomenclature to state the algorithm, disagreeing on a significant component of Neftçi's analysis, and adding several methods to evaluate the results.

They depart significantly from Neftçi around the idea of the *a priori probability*, which they term a *transition probability*, following the terminology of a Markov formulation.[3] They do not accept the idea that the transition probability changes over time, citing the results of prior work such as McCulloch, 1975. They observe:

> *In other work, we have presented evidence that the expansions and contractions in the American business cycle, particularly in*

2. Technically this formulation is a sequential-analytic stoppping-time framework whose properties are defined in Shiryaev, 1978.

3. See Diebold & Rudebusch, 1989 p 373.

the postwar period, are not characterized by duration dependence; thus, the probability of a turning point is roughly independent of the age of the regime. Diebold & Rudebusch, 1989 p. 376

They therefore substitute *fixed probabilities* for upward and downward transitions. The define these probabilities as Γ in the following formula which they set to .02 during expansions and which they set to .1 during contractions. They also make clear that this formulation is an application of Bayes formula. Their restatement has the following somewhat more compact form:

$$\Pi_t = [\Pi_{t-1} + \Gamma_t^u \cdot (1-\Pi_{t-1})] f^d(x_t \mid \bar{x}_{t-1}) / \{[\Pi_{t-1} + \Gamma_t^u \cdot (1-\Pi_{t-1})] f^d(x_t \mid \bar{x}_{t-1}) + (1-\Pi_{t-1}) f^u(x_t \mid \bar{x}_{t-1})(1-\Gamma_t^u)$$

Equation 2: D&R formula

On the subject of the transition probability, it should be noted that other authors follow Neftçi by stating:

..the likelihood of an imminent recession based on the length of the recovery to date compared with the average length of postwar recoveries...Historically, after 22 months into a recovery, the likelihood of a recession beginning in the very next month is only 2 percent, since postwar recoveries average much longer, 48 months. But after 73 months, the likelihood of a recession setting in immediately climbs to 10 percent, because a recession is overdue. In general, the formula's estimated probability will rise slightly in each successive month—apart from the new values of the indicator variable—as the recovery's life expectancy shortens. Palash & Radecki, 1985 p. 38.

A similar conclusion was reached by Ohn, Taylor, & Pagan, 2004, who find that both the business cycle and stock market cycle exhibit "duration dependence". In their nomenclature "positive duration dependence implies that new contractions are more robust to failure than more mature contractions" (p. 547) and while their results are somewhat inconclusive, they find strong evidence of such positive duration dependence in post-war economic expansions.

They are closer to Neftçi on the subject of the *conditional* probabilities, although they do not use Neftçi's idea of a smoothed empirical density, but instead propose the idea of fitting the regimes to a normal distribution. This paper follows this concept.[4] The data is divided manually into an "up" and a "down" regime, and each subset is fitted to a normal distribution. Figure 3 illustrates this fitting using the employment change data described in section **I.2**.

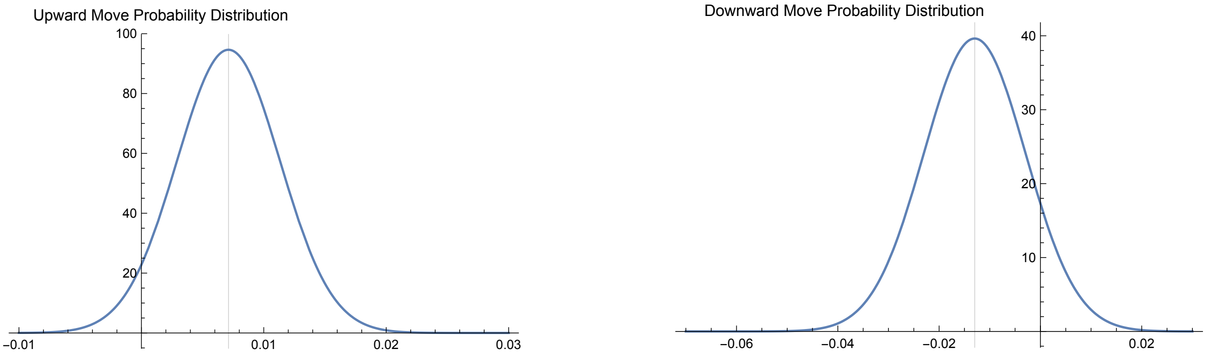

Figure 3: Upward and downward move probabilities

5. Transition Probability

In order to assess the issue of *transition* probability, this paper examines three concepts : a version of Neftçi's duration-dependent probability function; fixed probabilities (as per Diebold & Rudebusch); and a duration-dependent logistic function of the author's.

Neftçi's "Daniell window" approach[5] produces a discrete CDF for the probability of transition as the process evolves over time. Rather than directly replicating his approach (which is not intuitively obvious), we instead construct a conformal CDF through the interpolating function built in to Mathematica. This function produces identical values to Neftci's function at each discrete point in the data's evolution. This is illustrated in Figure 4, which shows the Mathematica interpolated function overlaid on Neftçi's[6].

Neftçi's function places the 50% probability point at around the 87th month, with 90% probability reached at approximately month 94.

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4. See Diebold & Rudebusch, 1989 p 377ff.

5. There seems to be a typographical error in the original article, with the window spelled "Daniel" rather than Daniell. The methodology of applying a smoothing window function seems clear, but the question of what smoothing function is applied to is not, at least to the author. Neftçi p. 236

6. The function was obtained by entering the values from Neftci's function manually, and then employing Mathematica's Interpolation function.

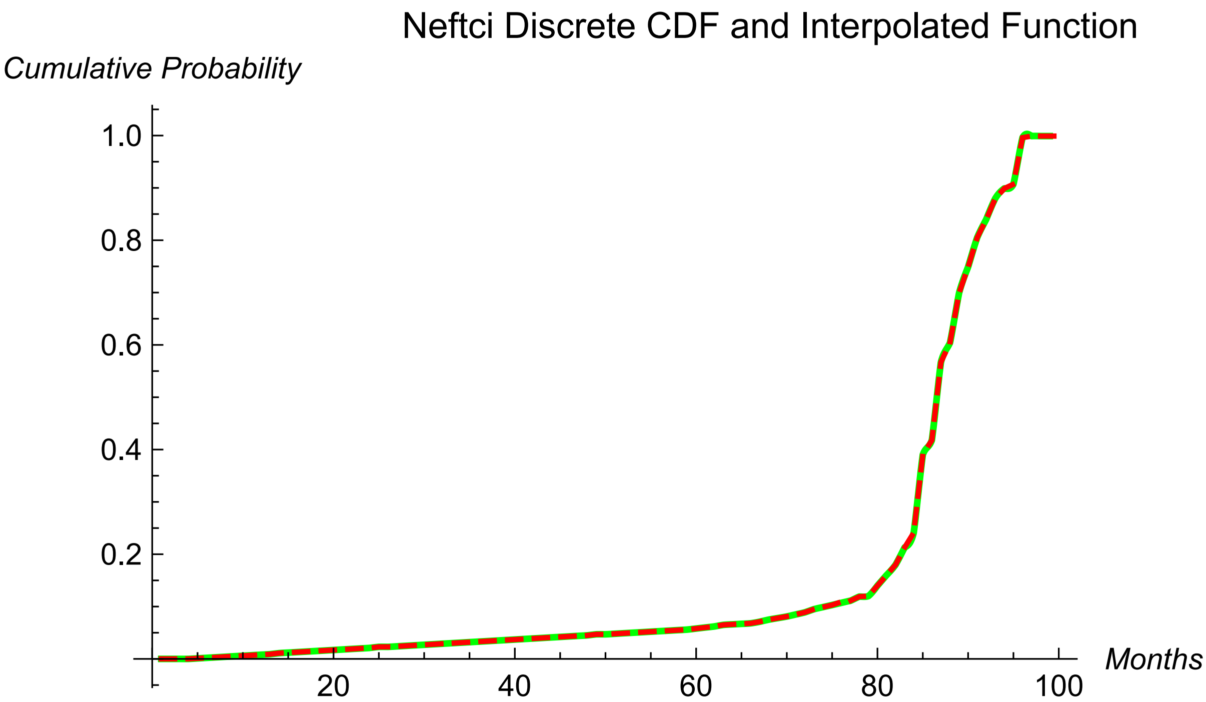

Figure 4: Neftçi transitional CDF

Use of Neftçi function does run the risk of norming the solution to the data, since a high transition probability drives the output of the **SPR** toward 1. Neftçi's approach seems to suffer from this quite a bit, since it ramps up the probability dramatically. One way to quantify this function is to look at the rate of change over an interval. Neftçi's cumulative probability function has a duration of 100 periods (months). From inception to period 74 his CDF accumulates less than 10% probability; from period 74 to 94 the' function increases the cumulative probability from 10% to 90%.

For purposes of comparison, this paper examines the effect of fixed probabilities for upward and downward regimes as well.

As an alternative to the Neftçi's transition probability function and to the fixed up or down transition probabilities of Diebold & Rudebusch, it is possible to construct a transition probability approach using a logistic function.[7] The parameters chosen provide the 50% probability at roughly the mean value of the empirical data for duration of the up and down regimes of the employment date. The two functions look like this:

7. One difference between the leading indicator series and the employment series used in this paper is that the leading indicator data is monthly and the employment data is quarterly. The paper adjusts for this difference.

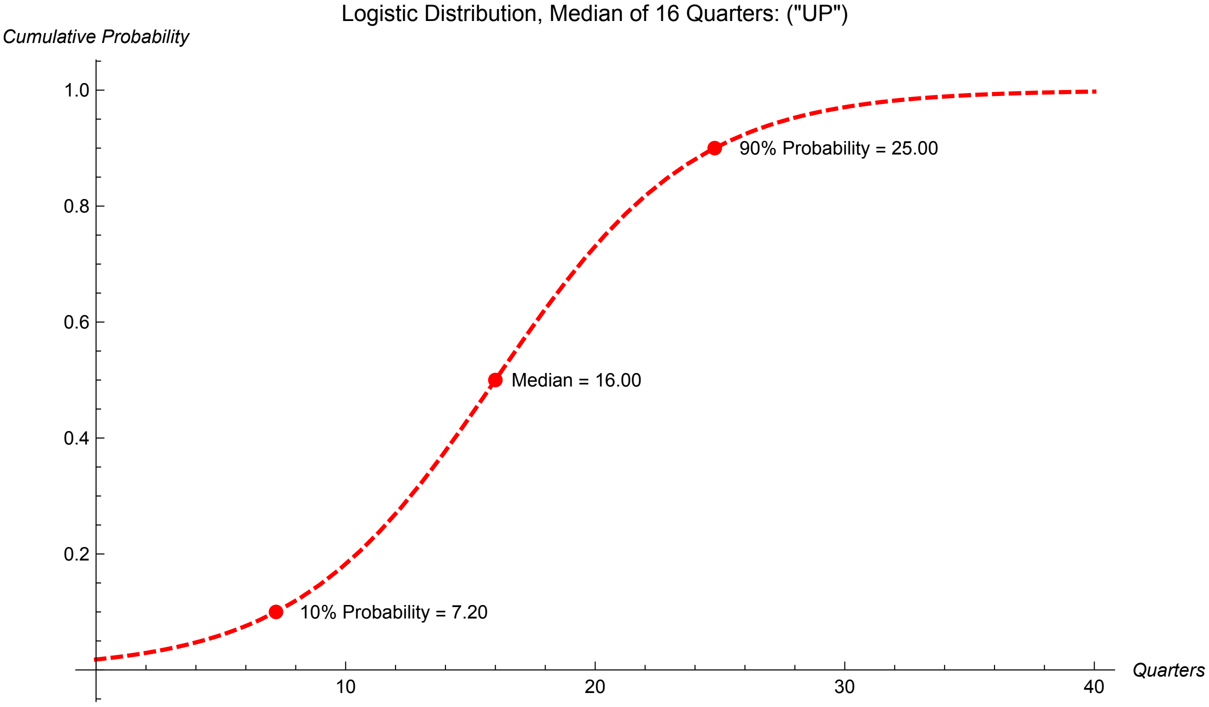

Figure 5: Logistic Up CDF

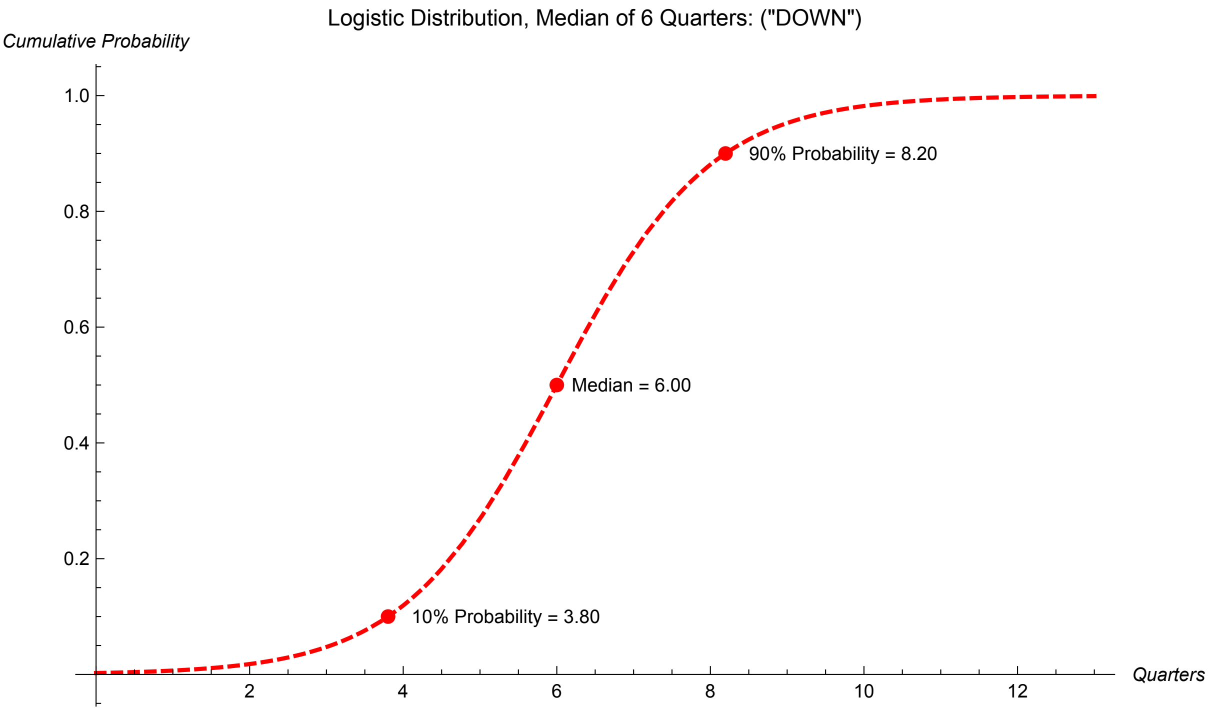

Figure 6: Logistic Down CDF

To adjust this function for use with quarterly data we multiply the period by four. The normalized logistic "UP" function has 10% cumulative probability by period 29, and it goes to period 100 before the cumulative probability reaches 90%. This broader range allows for a larger set of "UP" moves before the transition probability function drives the overall probability assessment close to 1.0.

6. Measures of Fit

D&R propose several measures of fit for the algorithm. The first measure is designed to assess the *accuracy* of the algorithm by comparing a set of 0/1 variables reflecting whether a turning point occured at a particular time t (or in a window) against the predicted probabilities. This is computed through a mean squared error formula:[8]

$$\mathbf{QPS} = \frac{1}{T}\sum_{t=1}^{T} 2\,(P_t - R_t)^2$$

Equation 3: QPS

In this formula, T is the number of periods, P_t is the predicted probability, and R_t is the realization (0 or 1). For the purpose of this paper, a three-period window around the realization variable is created, with the variable set to 1 in the period before and after the turning point, as well as the turning point period itself. As the author's observe, the QPS measure ranges from 0 to 2, with 0 being perfect in prediction.

D&R also propose a log version of the fit test (the log probability score):

$$\mathbf{LPS} = \frac{-1}{T}\sum_{t=1}^{T}[(1 - R_t)\ln(1 - P_t) + R_t \ln(P_t)]$$

Equation 4: LPS

This formulation runs from 0 to infininty, with a 0 indicating a perfect fit. This score penalizes large deviations more forcefully that the QPS.

These two measures are not statistical measures in the sense that they don't follow a known distribution, which prevents the computation of a confidence

8. See D&R p. 374

interval for the accuracy. Instead they provide a relative measure of the quality of fit between different components of the algorithm, or different leading indicator sets.

III. Summary of Results

1. Graphical Analysis of Fit

Figure 7 illustrates the evolution of the probability of transition at each point in the indicator series. The algorithm calls for turning points at the 19th, 30th, 49th, 56th, and 80th periods in the data.

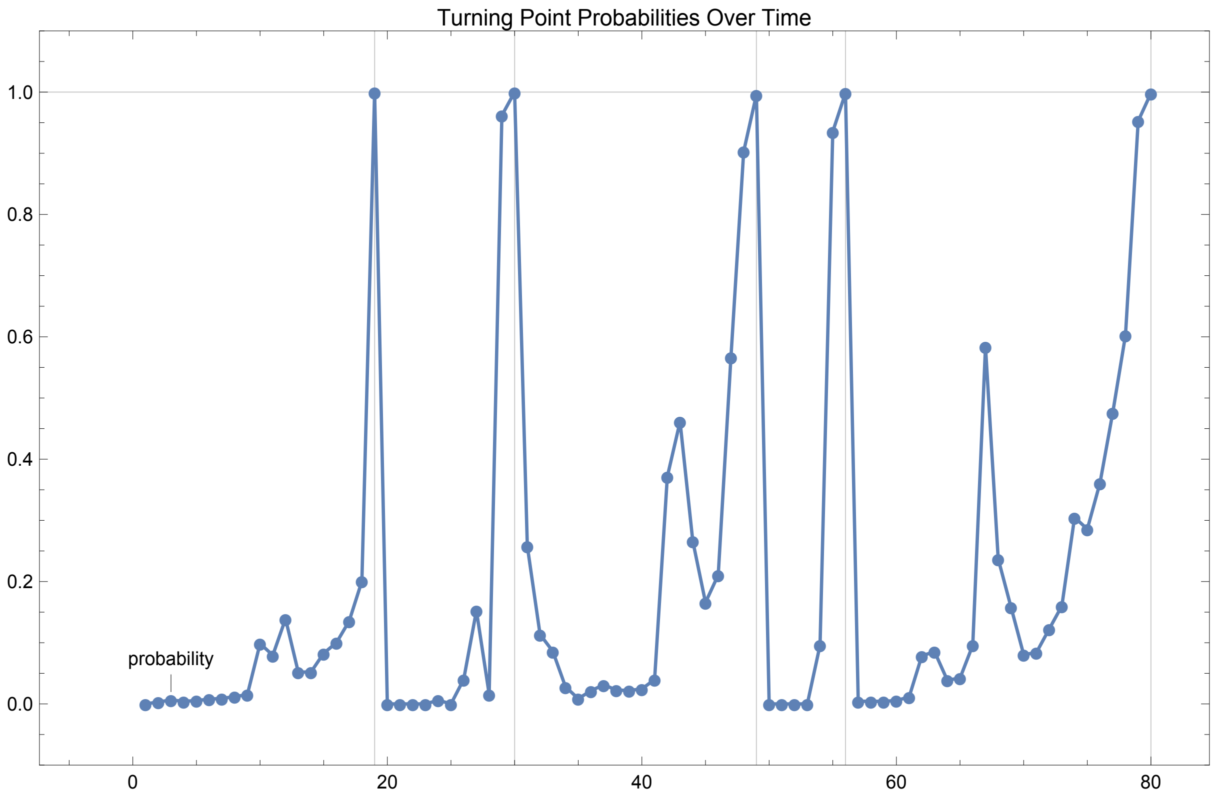

Figure 7: Turning point probabilities over time

Mapping these turning points into the rental data yields Figure 8. A visual survey of the graphics shows that the results were precise in the first downturn and second upturn; reasonable in the first upturn (especially given the slow-evolving trough), and slightly late in the second downturn.

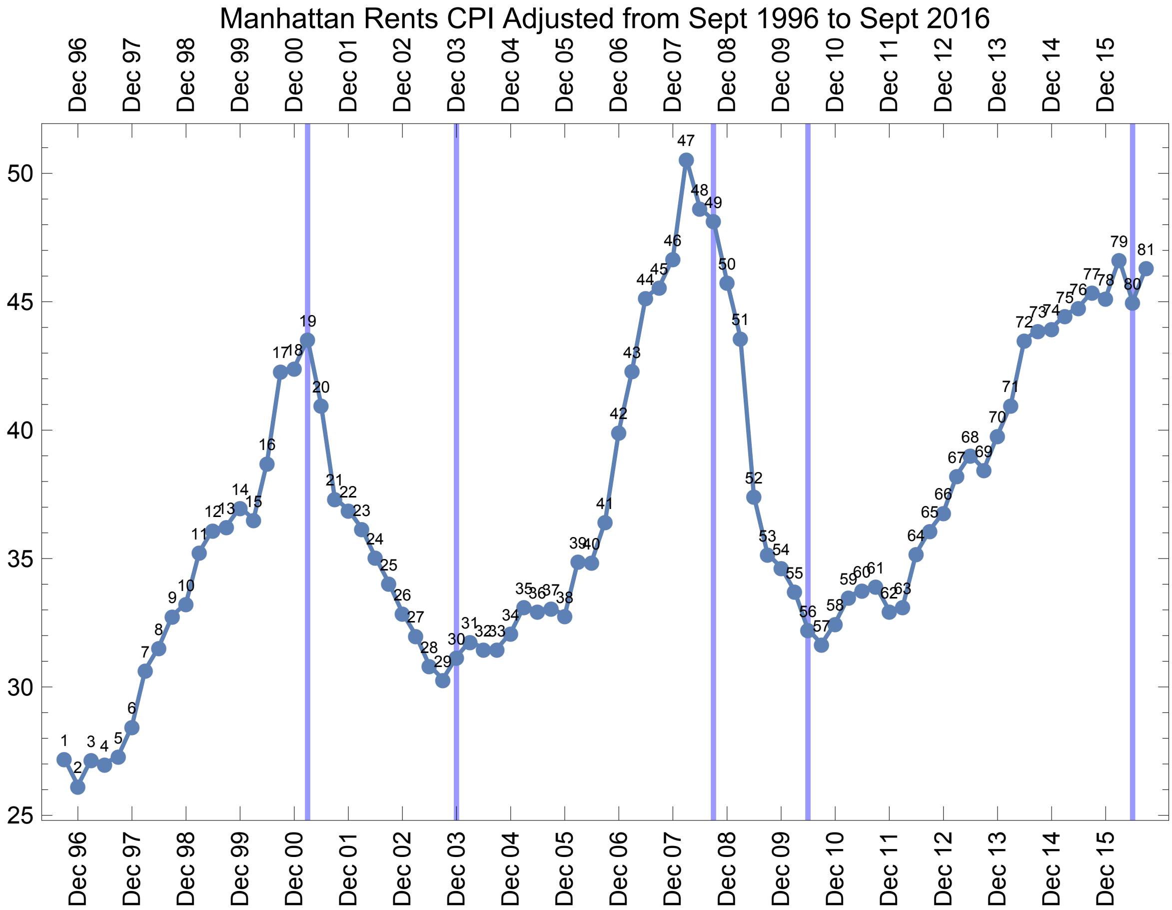

Figure 8: Indicated turning points

2. Quantitative Examination of Algorithm

The following table summarizes the values of D&R's LPS and QPS described in section **I.6** for each of the different transition probability approaches described in section **I.5**.

	QPS	LPS
Neftçi	0.36	1.27
Fixed	0.29	1.00
Logistic	0.28	0.86

With the data under consideration the Neftçi transitions work least well, and the logistic function works slightly better under the less punitive QPS test, and significantly better under the more stringent LPS test.

3. Numeric Results of SPR Runs

The following tables illustrate the time period (column 1), the prior probability (column 2), the up probability and the down probability in the current period (columns 3 and 4), and the SPR-computed turning-point probability. When the SPR probability goes above a predefined percentage, a turning point is called and the prior probability is reset to 0. In this analysis we employ a 99% probability. Each table ends with a forecasted turning point.

Table 1: First run

2	0.	0.0191897	0.18391	0.00314098
3	0.00314098	0.0282085	0.17997	0.00654733
4	0.00654733	0.0100093	0.125724	0.00449421
5	0.00449421	0.00617766	0.0740882	0.0056983
6	0.0056983	0.0155981	0.171179	0.00798061
7	0.00798061	0.00617766	0.0740882	0.00943998
8	0.00943998	0.012556	0.150795	0.0120233
9	0.0120233	0.012556	0.150795	0.0154172
10	0.0154172	0.0467361	0.115291	0.089432
11	0.089432	0.0337052	0.163923	0.0782481
12	0.0782481	0.0467361	0.115291	0.164021
13	0.164021	0.00790181	0.0992064	0.0571672
14	0.0571672	0.00617766	0.0740882	0.055442
15	0.055442	0.00366722	0.0350284	0.0846394
16	0.0846394	0.00478293	0.0523653	0.0976594
17	0.0976594	0.0191897	0.18391	0.137761
18	0.137761	0.00278453	0.0221756	0.206453
19	0.206453	0.156291	7.9001×10^{-7}	0.999998

Table 2: Second run

20	0.	0.122314	2.63145×10^{-10}	3.9404×10^{-11}
21	3.9404×10^{-11}	0.0855088	4.1773×10^{-13}	2.43221×10^{-13}
22	2.43221×10^{-13}	0.0261981	7.33941×10^{-20}	3.79143×10^{-19}
23	3.79143×10^{-19}	0.0674416	1.19331×10^{-14}	6.50928×10^{-14}
24	6.50928×10^{-14}	0.133635	0.000728419	0.00542126
25	0.00542126	0.149743	6.83288×10^{-8}	1.24962×10^{-6}
26	1.24962×10^{-6}	0.133635	0.000728419	0.038717
27	0.038717	0.125691	0.00154894	0.205079
28	0.205079	0.151501	0.0000544071	0.0241602
29	0.0241602	0.0892481	0.0182474	0.968848
30	0.968848	0.0542366	0.0890243	0.999953

Table 3: Third run

31	0.	0.0986536	0.0106983	0.217811
32	0.217811	0.0467361	0.115291	0.11737
33	0.11737	0.0542366	0.0890243	0.103445
34	0.103445	0.0337052	0.163923	0.0369663
35	0.0369663	0.0100093	0.125724	0.00974591
36	0.00974591	0.0337052	0.163923	0.0233504
37	0.0233504	0.0337052	0.163923	0.0323284
38	0.0323284	0.0191897	0.18391	0.0217406
39	0.0217406	0.0155981	0.171179	0.0223003
40	0.0223003	0.012556	0.150795	0.0256252
41	0.0256252	0.0191897	0.18391	0.0404352
42	0.0404352	0.0623314	0.0650592	0.338626
43	0.338626	0.000607303	0.000986624	0.467986
44	0.467986	0.0282085	0.17997	0.268644
45	0.268644	0.0233796	0.187006	0.178217
46	0.178217	0.0282085	0.17997	0.218075
47	0.218075	0.0542366	0.0890243	0.592587
48	0.592587	0.0709408	0.0449981	0.91293
49	0.91293	0.0892481	0.0182474	0.995121

Table 4: Fourth run

50	0.	0.14445	1.84951×10^{-8}	2.34511×10^{-9}
51	2.34511×10^{-9}	0.07631	7.25893×10^{-14}	4.73595×10^{-14}
52	4.73595×10^{-14}	0.0674416	1.19331×10^{-14}	2.39463×10^{-14}
53	2.39463×10^{-14}	0.137994	4.73674×10^{-9}	1.26277×10^{-8}
54	1.26277×10^{-8}	0.0986536	0.0106983	0.0978335
55	0.0978335	0.0337052	0.163923	0.9382
56	0.9382	0.0337052	0.163923	0.998476

Table 5: Fifth run

57	0.	0.0282085	0.17997	0.00471084
58	0.00471084	0.0191897	0.18391	0.00453813
59	0.00453813	0.012556	0.150795	0.00452345
60	0.00452345	0.012556	0.150795	0.0056929
61	0.0056929	0.0233796	0.187006	0.0109164
62	0.0109164	0.0542366	0.0890243	0.0668557
63	0.0668557	0.0467361	0.115291	0.0807435
64	0.0807435	0.0282085	0.17997	0.0415921
65	0.0415921	0.0282085	0.17997	0.0414966
66	0.0414966	0.039883	0.141309	0.0880759
67	0.0880759	0.0799577	0.029455	0.57578
68	0.57578	0.0233796	0.187006	0.235997
69	0.235997	0.0282085	0.17997	0.147376
70	0.147376	0.012556	0.150795	0.0829456
71	0.0829456	0.0100093	0.125724	0.0859353
72	0.0859353	0.0191897	0.18391	0.135236
73	0.135236	0.0191897	0.18391	0.177126
74	0.177126	0.0282085	0.17997	0.304097
75	0.304097	0.0155981	0.171179	0.28354
76	0.28354	0.0191897	0.18391	0.354678
77	0.354678	0.0233796	0.187006	0.483729
78	0.483729	0.0233796	0.187006	0.601701
79	0.601701	0.0623314	0.0650592	0.950547
80	0.950547	0.0799577	0.029455	0.998258

IV. Conclusions and Strategy

The results of this analysis need to be cast in terms of real estate strategy in order to be evaluated. In some circumstances, a tenant's actions do not allow them an option to time the market (given the nature of the search process and the time required to construct space). But a well organized search process--commenced early enough in the lease cycle--will afford optionality in the timing of lease negotiation and execution. How would this analysis have assisted a tenant who had the turning point predictions available in real time during the period under analysis?

In the case of the first turning point in 2000, if a tenant had the turning point indication (suggesting an imminent downturn) and had deferred lease execution for two quarters, rents would have dropped approximately 18%. That's a significant result over the course of a fifteen or twenty year transaction.

In the second turning point, the algorithm called the bottom in the last quarter of 2003, one period after the true bottom. While a failure to act immediately would not have had serious adverse consequence in this slowly-evolving trough, a decision to lease at this point or in the subsequent 2+ years would have positioned a tenant effectively to avoid the sharp increases from 2005 onwards.

The algorithm overshot the second peak in 2008 by two quarters (it's worst performance). However a decision to defer leasing even at this point would have reaped enormous financial benefit, as the global financial crisis caused a collapse in rents over the subsequent three quarters. Of course given the magnitude of the economic disruption it was not necessary to consult a computer oracle to know that waiting was the prudent strategy at that moment in the cycle!

Perhaps the most useful information from the algorithm would have been provided in the middle of 2010, when it signalled the beginning of a significant run-up that has persisted until the current time. A long-term lock-in at that point would have given optimal results (probably) over a ten-year period (unless another global financial crisis occurs in the next three years).

The most interesting output of the algorithm (and the one that remains to be known) involves the call of a turn at the end of 2016. While asset-side real estate analysts continue to forecast healthy rental growth, the jobs information seems to suggest that rents might be depreciating shortly.

Bibliography

Andersson, E., Bock, D., & Frisén, M. (2005). *Statistical surveillance of cyclical processes with application to turns in business cycles* (24(7)). John Wiley & Sons, Ltd. doi:10.1002/for.966

Chin, D. M., Geweke, J., & Miller, P. J. (2000). *Predicting turning points* ((267)). Federal Reserve Bank of Minneapolis. Retrieved from http://EconPapers.repec.org/RePEc:fip:fedmsr:267

Del Negro, M. (2001). Turn, turn, turn: Predicting turning points in economic activity. *Economic Review-Federal Reserve Bank of Atlanta*. Retrieved from http://search.proquest.com/openview/b0017d930bdb3623b2fd27b54283552c/1?pq-origsite=gscholar&cbl=34496

Diebold, F. X., & Rudebusch, G. D. (1989). Scoring the Leading Indicators. *The Journal of Business*, *62*(3), 369-391. Retrieved from http://EconPapers.repec.org/RePEc:ucp:jnlbus:v:62:y:1989:i:3:p:369-91

Frisen, M. (1994). Statistical surveillance of business cycles. *Research Report Department of Statistics Goteborg University*, *Research Report 1994:1*.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica: Journal of the Econometric Society*. Retrieved from http://citec.repec.org/d/ecm/emetrp/v_57_y_1989_i_2_p_357-84.html

McCulloch, J. H. (1975). *The Monte Carlo Cycle in Business Activity* (13(3)). Blackwell Publishing Ltd. doi:10.1111/j.1465-7295.1975.tb00251.x

McNees, S. (1992). How Large Are Economic Forecast Errors. *New England Economic Review*, *July/August*, 25-42.

Neftçi, S. N. (1982). Optimal prediction of cyclical downturns. *Journal of Economic Dynamics and Control*, *4*, 225 - 241. doi:http://dx.doi.org/10.1016/0165-1889(82)90014-8

Niemira, M. P. (1991). International application of Neftçi's probability approach. In K. M. Lahiri, G.H. (Ed.), *Leading economic indicators. New approaches and forecasting records* (pp. 91-108).

Ohn, J., Taylor, L. W., & Pagan, A. (2004). Testing for duration dependence in economic cycles. *The Econometrics Journal*. doi:10.1111/j.1368-423X.2004.00142.x

Palash, C. J., & Radecki, L. J. (1985). Using monetary and financial variables to predict cyclical downturns. *Economic Policy Review (Federal Reserve Bank of New York)*, 38-47.

Shiryaev, A. N. (1978). Optimal Stopping Rules, volume 8 of Applications of Mathematics. *Springer-Verlag, New York*.

Webb, R. H. (1991). On predicting the stage of the business cycle. In K. M. Lahiri, G.H. (Ed.), *Leading economic indicators. New approaches and forecasting records*. Cambridge: Cambridge University Press.

Zellner, A., Hong, C., & Min, C.-k. (1991). Forecasting turning points in international output growth rates using Bayesian exponentially weighted autoregression, time-varying parameter, and pooling techniques. *Journal of Econometrics*, *49*(1), 275 - 304. doi:http://dx.doi.org/10.1016/0304-4076(91)90016-7